NACA

RESEARCH MEMORANDUM

EFFECT OF CAPTURE ON THE SLOWING-DOWN LENGTH OF NEUTRONS IN HYDROGENOUS MIXTURES CONTAINING URANIUM

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EFFECT OF CAPTURE ON THE SLOWING-DOWN LENGTH OF NEUTRONS IN HYDROGENOUS MIXTURES CONTAINING URANIUM

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SUMMARY

A rigorous expression for the slowing-down length of neutrons in hydrogenous media has been derived by E. Fermi under the assumption that the nonhydrogen nuclei have infinite mass and that capture is absent. A generalization of Fermi's result which takes into account the presence of capture but still retains the assumption of infinite mass for the nonhydrogen nuclei has been obtained.

In order to obtain some estimate of the influence of capture on the slowing-down process, the two expressions have been applied to the same sample mixture, a uranium-235 and water mixture with the ratio of hydrogen to uranium nuclei chosen as 25. The results show that capture effected a 9.5-percent reduction in the calculated L_f^2, defined as the average slowing-down length squared at thermal energy of U-235 fission neutrons. A further calculation was made using the total mean free path in the no-capture formula. The value of L_f^2 obtained in this way differed by 3.9 percent from the value given by the capture formula.

THEORY

If the mass of the nonhydrogen nuclei in hydrogenous mixtures is assumed to be infinite, it is then possible to derive an otherwise rigorous expression for the slowing-down length. Suppose E_o is the energy of neutrons emitted from the monoenergetic point source, E is the neutron energy variable, and $E_b (E_b > E_o)$ is a base value for energy. Let $u = \log E_b/E$ and $u_o = \log E_b/E_o$. Further if $l(u)$, $l_c(u)$, and $l_H(u)$ are the total, capture, and hydrogen-scattering mean free paths, respectively, let $c(u) = l/l_H$ and $g(u) = l/l_c$. Then the slowing-down length L_s can be obtained as

$$L_s^2(u) = \frac{1}{3}\left\{\frac{l^2(u_o)}{c(u_o) + g(u_o)} + F(u) + \int_{u_o}^{u} du' \, \frac{c(u')}{c(u') + g(u')} F(u')\right\} \quad (1)$$

2851

where

$$F(u) = \frac{l^2(u)}{c(u) + g(u)} + l(u_o)\, l(u) \exp\left[-E(u)\right] +$$

$$l(u) \exp\left[-E(u)\right] \int_{u_o}^{u} du' \, \frac{l(u')\, c(u')}{c(u') + g(u')} \exp\left[E(u')\right]$$

and

$$E(u) = \int_{u_o}^{u} du' \left[\frac{3}{2} - c(u') - \frac{g(u')}{c(u') + g(u')}\right]$$

(These expressions are given in a communication to be published in the Physical Review.)

Setting $g = 0$ in equation (1) gives the no-capture result which has been derived by E. Fermi (ref. 1).

A calculation on a boron oxide and water mixture (ratio of hydrogen to boron nuclei chosen equal to 30) showed that L_f^2 was about 3 percent less when capture was taken into account. Capture by boron becomes important only in the epi-thermal energy region, so only for neutrons of low source energy does capture occur over an appreciable portion of the slowing-down energy range and thereby affect the thermal slowing-down length significantly. However, these neutrons have slowing-down lengths of relatively small magnitude and are weighted lightly in the uranium-235 fission spectrum, so that the change in L_f^2 due to including capture is small in the case of the boron oxide and water mixture. For the uranium-water mixture considered herein, there is significantly more high-energy capture present.

2882

Calculation Results for Uranium-Water Mixture

The mixture of uranium-235 and water chosen for the calculation is specified by a hydrogen to uranium nuclei ratio equal to 25. The calculated slowing-down lengths at thermal energy for various birth energy neutrons are given in table I $\left(\text{the birth energy } E_o \text{ is designated in terms of } u_o \equiv \log \frac{10^7}{E_o}\right)$. The first column of the table gives the results obtained by using the formula which includes capture. These are regarded as the correct values. The second and third columns give the results obtained from the no-capture formula in which the scattering mean free path and the total mean free path, respectively, have been used. It is seen that the third column agrees more closely throughout with the first column. The maximum discrepancies occur at $u_o = 5.8$ and amount to 32.6 percent in the second column and 11.0 percent in the third.

The average slowing-down lengths weighted over the fission spectrum are 21.12, 23.13, and 21.96, respectively, from the three columns. The second is 9.5 percent larger and the third is 3.9 percent larger than the first.

The expressions for the slowing-down length have the same form in each case and entail practically the same labor in computational use, so the general expression (1) is to be preferred whenever capture is present.

Lewis Flight Propulsion Laboratory
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REFERENCE

1. Marshak, Robert E.: Theory of the Slowing Down of Neutrons by Elastic Collisions with Atomic Nuclei. Rev. Modern Phys., vol. 19, no. 3, July 1947, pp. 185-238.

2861

TABLE I. - THERMAL SLOWING-DOWN LENGTH FOR VARIOUS

BIRTH ENERGIES

u_o	L_s^2 Including capture	L_s^2 No capture, scattering mean free path	L_s^2 No capture, total mean free path
0	129.99	143.13	136.89
.1	111.48	121.99	116.92
.2	97.57	106.29	102.03
.3	86.43	93.80	90.14
.4	75.28	81.43	78.34
.5	66.44	71.70	69.04
.6	58.73	63.26	60.93
.8	45.27	48.70	46.90
1.0	35.42	38.15	36.67
1.2	33.03	35.36	33.91
1.4	27.79	30.03	28.71
1.6	22.40	24.34	23.17
1.8	18.27	20.02	18.95
2.0	14.16	15.74	14.77
2.2	12.97	14.50	13.54
2.4	11.88	13.38	12.43
2.6	10.60	12.06	11.13
2.8	9.34	10.74	9.83
3.0	8.09	9.45	8.56
3.2	7.36	8.71	7.83
3.4	7.00	8.34	7.47
3.8	6.00	7.30	6.43
4.2	5.27	6.55	5.70
4.6	4.72	5.99	5.15
5.0	4.32	5.57	4.74
5.4	3.99	5.23	4.41
5.8	3.74	4.96	4.15